Microeconomics of Banking

Spring trimester, 2011 at the Toulouse School of Economics

These notes have been compiled in the hope they may serve as a useful study aid for students in the Microeconomics of Banking course at TSE. Greatful thanks to Nicolas who composed the first portion of this document, and with whom I have collaborated on this project. My grateful appreciation to professor Plantin, whose lectures are the basis for these notes, and for making his slides available. I would be glad to hear any comments or suggestions for improvement that may occur to the reader. All errors remain my own. - MJM
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0.1 Course Information


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Exam: The professor provided a few comments about the exam halfway through the lectures, saying it would be a two hour closed book examination. Though he hadn’t composed it at the date of his comments, he stated it would possibly contain one essay type question, to check that we can make use of the models we studied in class to make arguments, and an additional one or two problems regarding the type of models we saw in class.

0.2 Introduction

Definition. (Bank) a Bank is an institution whose current operations consist in granting loans and receiving deposits from the public. Let’s break down this definition, to be as precise as possible.

- "Current": current operations vs. occasional operations
- "And": combination of lending and borrowing is typical characteristics of commercial banks
- "Public": on the liabilities side of the bank, the customers are general public which is not armed to assess the safety and soundness of financial institutions. The following schematic shows the components of the economy we consider in our models:
In studying the microeconomics of banking, we shall make a distinction between two sources of demand for funds. Broadly, it is possible to distinguish 'banks' from 'markets' as places which attract the funds supplied by households.

First a word about markets - these can be further separated into centralized and decentralized markets. Most markets attracting funds from lenders, savers or the general public are decentralized. Decentralized markets or over-the-counter (OTC) markets include most derivatives markets, corporate bond markets, and the foreign exchange (Forex) market, while centralized markets include most stock markets and some derivatives markets like Chicago Mercantile Exchange. Our discussion will begin however, with banks, of which there are two main types we study:

- **Commercial banks**
  - These issue loans which are non-contingent on a specific project (as opposed to equity); they typically are large and long-term assets. From the bank's point of view (and as reflected on their balance sheet), these loans are assets whose cashflows are the payment streams of interest and principal associated to them.
  - The bank's liabilities are the deposits of their customers. Deposits are contracts where the depositors lend money to the bank, but with a liquidity option which allows them to withdraw money easily. They are typically small (relative to loans), and unlike the loans banks make, which tend to have a well-specified set of repayment times, the liquidity feature means that they bank needs to be ready to repay depositors the money they've lent at any time.
  
  * Thus, there is a discrepancy between the terms of the contracts, which means the bank has risk.
Some (ex. Friedman) voice the opinion that this discrepancy is an historical accident.

Note: It could be quite costly for a single, small depositor to monitor the solvency of the bank where they make their deposit, and would cause free-riding problems if some, but not all, people were monitoring.

• Investment banks

  – Advise firms issuing bonds and equity
  – Advise banks wishing to securitize a portfolio of loans
  – Generally, help people to gain access to the financial markets. For example, they could supervise, for insurance reasons, and monitor a firm wishing to make an IPO

0.2.1 Functions of Banks:

1. Offering liquidity and payment services (this includes automatic teller machines, providing checkbooks, etc.)

2. Transforming assets

   a) Convenience of denomination (amount)
   b) Quality transformation
   c) Maturity (duration) transformation

3. Managing risks

   a) Credit risk
   b) Interest rate and liquidity risk

4. Processing information and monitoring borrowers

0.2.1.1 Offering liquidity and payment services

• Here, a slight historical detour. In the past, banks played two different parts in the management of fiat money: money changing and provision of payment services

  – Money changing

    * Greece: Trapeza (Greek word for bank): means 'balance' that was used by early money changers to weigh coins
    * Italy: Banco (Italian word for bank): bench on which the money changers placed their precious coins
    * Management of deposits
· Safekeeping services: initially, bank deposits were not supposed to be lent and they had a zero (or even negative, to compensate the bank for their expenses) return
· Convertible into "good money": Coins differed in their composition of precious metals and banks are required to make payment in good money

* Payment services
  · They cover both the management of clients' accounts and the finality of payments
  · The safety and efficiency of these payment systems have become a fundamental concern for governments and central banks

0.2.1.2 Transforming assets

* Convenience of denomination
  – Banks choose the unit size (denomination) of its products (deposits and loans) in a way that is convenient for its clients
    · Typically collect small deposits and invest the proceeds into large loans
  – Quality transformation
    · Bank deposits offer better risk-return characteristics than direct investments. Reasons:
      · Indivisibilities in the investment: small investors cannot diversify their portfolio
      · Asymmetric information: banks have better information than depositors about the entrepreneurs and firms they finance
  – Maturity transformation
    · Transforming securities with short maturities into securities with long maturities (accepting deposits and making loans)
    · Origin of interest rate and liquidity risks

0.2.1.3 Managing Risk

* Credit risk
  – That is the risk that a borrower is not able to repay a debt (principal or interest)
    · Banks tried to make their loans secure either through collateral, through the assignment of rights or through the endorsement by a city
Commercial banks are doing tranching, which means they take the first losses. When losses occur, it is sustained by the capital and, when there is no capital remaining, it then affects the availability of deposits.

- Interest rate and Liquidity risks

* Maturity transformation exposes banks to risks
  - Cost of funds (which depends on the level of short-term interest rates) may rise above the contractual interest rates of the loans granted by the banks
  - Unexpected withdrawals of deposits may force banks to seek more expensive sources of funds

⇒ Banks have to manage the combination of interest rate risk (due to the difference in maturity) and liquidity risk (due to the difference in the marketability of the claims issued and of the claims held)

- The internal rate of return a bank asks on her loans should be linked to the weighted cost of capital from deposits and equity.

0.2.1.4 Monitoring and Information Processing

- Banks screen loan applicants and monitor the way the businesses they finance manage their projects. Through these activities, banks and their borrowers develop long-term relationships, thus mitigating the effects of moral hazard

  - This is one of main differences between bank lending and issuing securities in the financial markets. One way of thinking about the value of a bank loan, over and above direct lending, is that which results from the long-term relationship. We can also note that the loan value is *a priori* unknown to outsiders: bank loans are "opaque"

- Securitization may induce some problems, since the bank may not have the incentive to maintain their role of monitoring loans. They create short-term instruments and sell them, hence the long-term relationship is further transformed in a short-term one.
U.S. relies more on stock markets and the Euro area relies more on the banking system

0.2.2 Resource allocation

Some general comments on how banks play a role in the way resources are allocated in an economy. Banks exert a fundamental influence on capital allocation, risk sharing and economic growth.

- Underdeveloped economies with a low level of financial intermediation and small, illiquid financial markets may be unable to channel savings efficiently.

- Market-oriented economies are not very good at dealing with nondiversifiable risks.
• Banks’ reserves function as a buffer against macroeconomic shocks and allow for better intertemporal risk sharing

• Bank-oriented economies are not very good at financing new technologies. Markets are much better for dealing with differences of opinion among investors about these new technologies

• Market-oriented economies are better at innovating, since there are more people looking at the innovating sectors and offering opinion, hence the information is more accurate.

0.2.3 Banking in the Arrow-Debreu Model

• The economy:
  
  - Two dates: $t = 1; 2$
  - Unique physical good initially owned by consumers
  - Agents are households ($h$), firms ($f$), and banks ($b$). All agents behave competitively.

  
  
  Consumers:
  
  - Choose date 1 and 2 consumption ($C_1, C_2$) and the allocation of savings, which is split between a deposit $D$ in a bank and bonds $B$
held in the market for firm debt \((D_h, B_h)\) such that

\[
\begin{align*}
\max & \quad U(C_1, C_2) \\
\text{s.t.} & \quad C_1 + D_h + B_h = \omega_1 \\
& \quad \rho C_2 = \Pi_f + \Pi_b + (1 + r)B_h + (1 + r_D)D_h,
\end{align*}
\]

where \(\rho\) is the level of inflation, \(r\) is the time 2 return offered by banks to investors, \(D_h\) is the amount of deposit made by the household, and \(B_h\) is the amount in bonds held by the household. In words, the above says that consumers maximize their total utility from period one and period 2 such that all their initial wealth at time 1 is split between consumption, bank deposits, and lending to the bond market, and that their consumption at time 2 equals their inflation-adjusted payments from their ownership of firms and banks plus the repayment of the principal and interest on their bonds plus the principal and interest on their bank deposits.

- Interior solution: \(r = r_D\); if there is no friction this condition is required for the market to clear

- Firms choose investment level \(I\) and its financing from bank loans \(L\) and bonds \((L_f, B_f)\) such that they maximize their profits

\[
\begin{align*}
\max & \quad \Pi_f \\
\text{s.t.} & \quad \Pi_f = pf(I) - (1 + r)B_f - (1 + r_L)L_f, \\
& \quad I = B_f + L_f
\end{align*}
\]

which yields an interior solution: \(r = r_L\) by market clearing condition.

- Banks choose loan supply \(L_b\) and its financing \((D_b, B_b)\) to maximize their profits

\[
\begin{align*}
\max & \quad \Pi_b \\
\text{s.t.} & \quad \Pi_b = r_LL_b - rB_b - r_DD_b, \\
& \quad L_b = B_b + D_b
\end{align*}
\]

- General equilibrium is characterized by a vector of interest rates \((r, r_L, r_D)\) and three vectors of demand and supply levels: \((C_1, C_2, B_h, D_h)\) for the consumer, \((I, B_f, L_f)\) for the firm and \((L_b, B_b, D_b)\) for the bank such that

- Each agent behaves optimally, and
- Each market clears:
  - \(I = S\) (goods market)
  - \(D_b = D_h\) (deposit market)
  - \(L_f = L_b\) (loan market)
  - \(B_h = B_f + B_b\) (bond market)
**Proposition.** If firms and households have unrestricted access to perfect financial markets, then in a competitive equilibrium:

- **Banks make zero profit**
- The size and composition of banks’ balance sheets have no effect on other economic agents
  - The markets clear with \( r = r_D = r_L \)
Part I

The Role of Financial Intermediaries (FIs)
0.3 Introduction

**Definition.** A Financial Intermediary (FI) is an economic agent who specializes in the activities of buying and selling (at the same time) financial claims. We will usually generically refer to financial intermediaries as 'banks' for modeling purposes.

- Banks usually deal with financial contracts (loans and deposits) which cannot be easily resold (are less liquid), as opposed to financial securities (stocks and bonds) that are marketable instruments.
  - Securitization allows banks to resell the loans they originated. However, asymmetric information may limit the possibilities of securitization.
  - Banks have to transform financial contracts and securities because the contracts and securities issued by firms (borrowers) are usually different from those desired by investors (depositors); difference in duration and size. In addition, we have seen in the course on Corporate Finance that the less-marketable contracts offered by banks tend to have features such as covenants that may require the lender to maintain a relationship with the borrower, for monitoring purposes, for example. If debtholders are broadly dispersed, these kind of contracts will be hard to enforce because of free-rider problems, etc.

- Existence of financial institutions:
  - Classical transaction cost justifications
    * Existence of economies of scale and economies of scope
    * Physical and technological costs do not provide a satisfactory justification given significant progress in telecommunications and computers which we might have expect to have radically lowered such costs.
  - Informational Asymmetries
    * These asymmetries generate market imperfections that can be seen as specific forms of transaction costs
      - Banks gather information by accepting deposits and making loans, hence they have more information on the liquidity provision; they have more information on risk.

- Classical Theory:
  - Do economies of scope exist between deposit and credit activities?
    * "Central place" story
Because of transportation costs, it is efficient for the same firm or branch to offer deposit and credit services in a single location.

* Portfolio theory

- In equilibrium, less risk-averse investors short-sell (borrow) the riskless asset and invest more than their own wealth in the risky market portfolio.
- Diversification argument (Pyle 1971): Banks are interpreted as investors who hold a long position in securities having a positive expected excess return and a short position in securities that have a negative expected excess return under the assumption that the returns of these two categories of securities are positively correlated.

**Pyle:**

- The agent (Bank) have mean-variance preferences defined by: \( \mathbb{E}[\tilde{R}] - \mathbb{V}(\tilde{R}) \)
- Let’s have 3 assets:

<table>
<thead>
<tr>
<th>Asset</th>
<th>return</th>
<th>variance</th>
</tr>
</thead>
<tbody>
<tr>
<td>Risk-free</td>
<td>( r )</td>
<td>0</td>
</tr>
<tr>
<td>Deposit</td>
<td>( r_D )</td>
<td>( \sigma_D^2 )</td>
</tr>
<tr>
<td>Loan</td>
<td>( r_L )</td>
<td>( \sigma_L^2 )</td>
</tr>
</tbody>
</table>

- The correlation between (the return on?) deposits and loans being \( \rho \)
- Let \( \omega_D \) and \( \omega_L \) be the proportions of an agent’s total wealth \( \omega \) of deposits and loans respectively. The returns are given by

\[
\tilde{R} = (\omega - \omega_D - \omega_L) \cdot r + \omega_D \tilde{r}_D + \omega_L \tilde{r}_L,
\]

\[
\mathbb{E}[\tilde{R}] = (\omega - \omega_D - \omega_L) \cdot r + \omega_D r_D + \omega_L r_L
\]

in expectation, and the variance is

\[
\mathbb{V}(\tilde{R}) = \omega_D^2 \sigma_D^2 + \omega_L^2 \sigma_L^2 + 2 \rho \omega_D \omega_L \sigma_D \sigma_L.
\]

- The utility function (using the form of mean-variance preferences stated above) can then be written as:

\[
U = \omega_D (r_D - r) + \omega_L (r_L - r) - \omega_D^2 \sigma_D^2 - \omega_L^2 \sigma_L^2 - 2 \rho \omega_D \omega_L \sigma_D \sigma_L
\]

Maximizing this expression with respect to the share of wealth invested in deposits and loans, the first order conditions (F.O.C.s) are

\[
\frac{\partial U}{\partial \omega_D} = 0 \quad \Rightarrow \quad r_D - r = 2 \omega_D \sigma_D^2 + 2 \omega_L \sigma_D \sigma_L,
\]
\[ \frac{\partial U}{\partial \omega} = 0 \implies r_L - r = 2\omega_L \sigma_L^2 + 2\rho \omega_D \sigma_D \sigma_L, \]

which is a system of two equations in two unknowns

\[ \begin{bmatrix} 2\sigma_D^2 & 2\rho \sigma_D \sigma_L \\ 2\rho \sigma_D \sigma_L & 2\sigma_L^2 \end{bmatrix} \begin{bmatrix} \omega_D \\ \omega_L \end{bmatrix} = \begin{bmatrix} r_D - r \\ r_L - r \end{bmatrix} \]

yielding:

\[ \omega_D = \frac{2(r_D - r)\sigma_D^2 - 2(r_L - r)\rho \sigma_D \sigma_L}{4\sigma_D^2 \sigma_L^2 (1 - \rho^2)} \quad \omega_L = \frac{2(r_L - r)\sigma_D^2 - 2(r_D - r)\rho \sigma_D \sigma_L}{4\sigma_D^2 \sigma_L^2 (1 - \rho^2)} \]

If \( r_D < r < r_L \) and \( \rho \geq 0 \), then we have \( \omega_D < 0 \) and \( \omega_L \geq 0 \)

- The difficulty is to show how to get \( r_D < r < r_L \) and \( \rho \geq 0 \), the theory does not explain where these assumptions come from.

- Economies of Scale
  - The presence of fixed transaction costs (e.g. fixed fee, indivisibilities)

- Liquidity insurance provision
  - Bryant (1980) and Diamond and Dybvig (1983). Depository institutions are considered as pools of liquidity that provide households with insurance against idiosyncratic shocks that affect their consumption needs

### 0.4 Diamond and Dybvig

- **Model:**
  - An economy with three dates (\( t = 0, 1, 2 \)) and one good that is to be consumed at \( t = 1 \) or \( t = 2 \)
    * liquidity risk is modelled as uncertainty on the consumption date
  - A continuum of ex-ante identical agents. These will end up being one of two types, but at the beginning of the model, agents have not yet learned their type. Each is endowed with one unit of good at \( t = 0 \)
  - At \( t = 1 \), agents learn their types. There are two possibilities:
    * Impatient agents (type 1): need to consume early \( \implies \) Utility is \( u(C_1) \). The idea is that type one agents only get utility from consumption in period 1. Thus we could have written their utility as \( u(C_1) + 0(C_2) \).
Likewise, the patient agents (type 2): only get utility from consuming late $\implies$ Utility is $u(C_2)$ (the utility derived from consumption in period 2 for the impatient agent and period 1 for the patient agent is 0.)

* At time 1, when the agent learns her type, this is her private information.

- At $t = 0$, the probability of being type i ($i = 1; 2$) is $\pi_i$: Thus we can write the ex-ante expected utility of each agent as

$$U = \pi_1 \cdot u(C_1) + \pi_2 \cdot u(C_2).$$

Assumptions:

* $u''(C) < 0 < u'(C)$ (the utility function is increasing and concave)
  - No discounting (to simplify the exposition; we added this without loss of generality)

- Two investment technologies

  * Storage technology (liquid). For example a bank deposit - this good can be stored from one period to the next
  * Long-run technology (illiquid). This is more like a loan

<table>
<thead>
<tr>
<th></th>
<th>$t = 0$</th>
<th>$t = 1$</th>
<th>$t = 2$</th>
</tr>
</thead>
<tbody>
<tr>
<td>Risk-free</td>
<td>invest 1</td>
<td>get 1 if consume at $t=1$</td>
<td>get 1 if consume at $t=2$</td>
</tr>
<tr>
<td>Long-run</td>
<td>invest 1</td>
<td>early liquidation, obtain $i &lt; 1$</td>
<td>obtain $R &gt; 1$</td>
</tr>
</tbody>
</table>

- Note: this is not a full theory of banking as there is no aggregate risk (by a law of large numbers since there is a continuum of agents?).

**Optimal Allocation**

- To obtain Pareto-optimal allocation, we have to find $I$ (amount of investment in illiquid technology) such that

$$\begin{cases} 
\text{Max} & \pi_1 \cdot u(C_1) + \pi_2 \cdot u(C_2) \\
\text{s.t.} & \pi_1 \cdot C_1 = 1 - I \\
& \pi_2 \cdot C_2 = R \cdot I 
\end{cases}$$

We can combine the constraints: $\pi_1 C_1 + \frac{\pi_2 C_2}{R} = 1$

**Proposition.** The optimal allocation $(C_1^*, C_2^*)$ satisfies the following condition $u_0(C_1) = R \cdot u_0(C_2)$

- Where with $R \geq 1$, $u'' < 0 \implies C_2^* \geq C_1^*$
If \( l = 1 \), then there is no cost of liquidating the short-term technology relative to the short-term, hence everyone invests in the long-term vehicle and the first best is achieved.

- In this model, the presence of private information does not matter: the first best is incentive compatible.
  - * The patient type does not want to mimic the impatient type since \( C_2^* \geq C_1^* \)
  - * The impatient type wants to consume now
- If we allow each agent to lend and borrow to each other, then there can be an incentive problem

**Allocation in Autarky**

- What is the allocation obtained when there is no trade between agents (i.e. in autarky)?
- Each agent has to solve:

\[
\begin{align*}
\max & \quad \pi_1 \cdot u(C_1) + \pi_2 \cdot u(C_2) \\
\text{s.t.} & \quad C_1 = 1 - I + l \cdot I \\
& \quad C_2 = 1 - I + R \cdot I
\end{align*}
\]

**Proposition.** *The autarky allocation is inefficient because \( \pi_1 \cdot C_1 + \frac{\pi_2 \cdot C_2}{R} < 1 \)*

- We have \( C_1 < 1 \) (unless \( I = 0 \)) and \( C_2 < R \) (unless \( I = 1 \)) which yield \( \pi_1 \cdot C_1 + \frac{\pi_2 \cdot C_2}{R} < 1 \)

**Market Allocation:**

- A bond market is open at \( t = 1 \) allowing agents to trade
  - * \( p \) is the price at \( t = 1 \) of the bond that yield one unit of good at \( t = 2 \)
  - \( p < 1 \), otherwise the market does not clear; no one would be willing to buy the asset.
- In the presence of bond market, each agent has to solve

\[
\begin{align*}
\max & \quad \pi_1 \cdot u(C_1) + \pi_2 \cdot u(C_2) \\
\text{s.t.} & \quad C_1 = 1 - I + pRI \\
& \quad C_2 = RI + \frac{1-I}{p}
\end{align*}
\]

- Interior equilibrium price satisfies \( p \cdot R = 1 \); otherwise there is an arbitrage opportunity on the market. If \( p \cdot R < 1 \) (\( p \cdot R \geq 1 \)) everyone would be willing to buy(sell), but no one would be willing to sell(buy).
More precisely, one can constitute a portfolio with a long position on the long-term asset and a short position on the short-term asset, this portfolio would have a payment of 1 in the first period and -R in the second. An agent can then sell the portfolio (if \( p \cdot R < 1 \)) and buy the bond in the first period and make a profit of \( 1 - p \cdot R \).

- Equilibrium allocation of the market economy

- \( C_M^1 = 1 \)
- \( C_M^2 = R \)
- \( I^M = \pi_2 \)

  - By the market clearing condition: \( \pi_1 RI = (1 - \pi_1)(1 - I) \cdot R \)
  - Idea: Impatient type sell his proportion of the long-term asset in order to consume in period 1 and the amount must be equal to the patient type remaining resources \( (1 - \pi_1) \cdot (1 - I) \) (remember that by absence of arbitrage, we have \( p = \frac{1}{R} \)).

- Under the assumption that \( -\frac{Cu''(C)}{u'(C)} \geq 1 \) (i.e. the function \( C \mapsto Cu'(C) \) is decreasing), we have \( Ru'(C_M^2) < u'(C_M^1) \) that means, the market allocation is not optimal (the FB was \( u'(C_M^1) = Ru'(C_M^2) \)).

- Incomplete market would explain the inefficiency, there is no private information assumed. Incomplete market because people are exposed to risk they cannot hedge.

- Financial Intermediation:

  - The Pareto-Optimal allocation \((C_M^1, C_M^2)\) can be implemented by a financial intermediary (FI) who offers a deposit contract stipulating that in exchange for a deposit of one unit at \( t = 0 \), individuals can get either \( C_M^1 \) at \( t = 1 \) or \( C_M^2 \) at \( t = 2 \). To fulfill its obligations, the FI stores \( \pi_1 \cdot C_M^1 \) and invest \( I = 1 - \pi_1 \cdot C_M^1 \) in the illiquid technology.

**Proposition.** In an economy in which agents are individually subject to independent liquidity shocks, the market allocation can be improved by a deposit contract offered by a financial intermediary.

  - Remark: In this simple setup, an FI cannot coexist with a financial market.

- Idea:
- Banks can insure the agent against shocks
- If there is only one bank in the economy but no market, then the FB can be achieve and the private information does not matter. There is no profitable deviation, reporting the other type is not incentive compatible
- Financial institutions and markets cannot coexist, otherwise non-truthful reporting can be IC
- First come, first serve rule: a run equilibrium can exist, since if agent anticipate a run at date 1, it is rational to run in order to avoid being stuck without money at date 2.

- **Information Sharing Coalitions**
  - In the presence of adverse selection, there exists scale economies in the borrowing-lending activity
  - Leland and Pyle (1977):
    - A coalition of borrowers, interpreted as FI, is able to obtain better financing conditions than an individual borrower

0.5 Leland and Pyle

0.5.1 Model of Capital Market with Adverse Selection

- A large number of entrepreneurs
  - Each has initial wealth $W_0$ and is endowed with a risky project of size normalized to 1.
    - Gross return of the project $\tilde{R}(\theta) = 1 + \tilde{r}(\theta)$ where $\tilde{r}(\theta) \sim N(\theta, \sigma^2)$
    - $\theta$ differs across projects - we can think of it as an entrepreneur’s type
    - The entrepreneurs have an exponential Von Neumann-Morgenstern utility function $u(w) = -e^{-\rho w}$
    - Because of risk aversion, even $W_0 > 1$, the entrepreneurs would prefer to sell their projects.

- Investors:
  - Risk-neutral
    - Access to a costless storage technology

- $\theta$ is observable
  - Price of the project is contingent on $\theta$, $P(\theta) = E[\tilde{r}(\theta)] = \theta$
* Each entrepreneur would sell his project at that price and be perfectly insured

- \( \theta \) is **privately observed** by the entrepreneur (adverse selection)

  - The price \( P \) is the same for all firms
  
  * If self-financed, each entrepreneur obtains:
    
    \[
    \mathbb{E}[u(W_0 + \hat{r}(\theta))] = u(W_0 + \theta - \frac{1}{2} \rho \sigma^2)
    \]
    
    (1)
  
  * If selling the project to the market, he obtains \( u(W_0 + P) \)
  
  * Cutoff level for \( \hat{\theta} \) is \( \hat{\theta} = P + \frac{1}{2} \rho \sigma^2 \)

**Proposition.** *Only those entrepreneurs with a relatively low expected return \( \theta < \hat{\theta} \) will sell the project. Hence, the equilibrium price will be \( P = \mathbb{E}[\theta | \theta < \hat{\theta}] \)*

- **Example:** binomial distribution of \( \theta \):

  \[
  \theta = \begin{cases} 
  \theta_1 & \text{with probability } \pi_1 \\
  \theta_2 & \text{with probability } \pi_2 
  \end{cases}
  \]

  where \( \theta_1 < \theta_2 \)

  - An efficient equilibrium where all entrepreneurs sell their projects means that \( \hat{\theta} \geq \theta_2 \). In that case \( P = \mathbb{E}[\theta] = \pi_1 \cdot \theta_1 + \pi_2 \cdot \theta_2 \). So, given (1), it is possible if:

    \[
    \pi_1 \cdot \theta_1 + \pi_2 \cdot \theta_2 + \frac{1}{2} \rho \sigma^2
    \]

    or

    \[
    \pi_1 (\theta_2 - \theta_1) \leq \frac{1}{2} \rho \sigma^2
    \]

    (2)

  * If (2) is not satisfied, at the equilibrium, some entrepreneurs prefer to self-finance \( \implies \) inefficient equilibrium

- **Idea:**

  * If there are too many bad entrepreneurs, then the discount is too great and the good entrepreneurs would not be willing to sell their project. The outcome is inefficient since some risk-averse entrepreneurs keep the risk they are facing.

  * When all good entrepreneur trade, then it is efficient since no risk-averse agent hold risk. The good entrepreneur subsidize the bad one.
0.5.2 Signalling

- When (2) is not satisfied, we have an inefficient equilibrium where good-quality projects are self-financed.

  - Leland and Pyle (1977): Good entrepreneurs can signal the quality of their projects by investing their own wealth into the project

    * Let \( \alpha \) be the fraction of the project self-financed by the good entrepreneur; hence he sells a fraction \( 1 - \alpha \)

      - Idea: If the entrepreneur is of the good type, then he should take more debt than equity; equity being equivalent to selling the project, hence getting rid of the risk.

    * No mimicking condition

      \[
      u(W_0 + \theta_1) \geq u(W_0 + (1 - \alpha)\theta_2 + \alpha \hat{r}(\theta_1))
      \]  

    * In case of normal distribution and exponential utility, the condition (3) become

      \[
      \frac{\alpha^2}{1 - \alpha} \geq \frac{2(\theta_2 - \theta_1)}{\rho \sigma^2}
      \]  

      \( (4) \) comes from then fact that in (3) we have:

      \[
      u(W_0 + \theta_1) \geq u(W_0 + (1 - \alpha)\theta_2 + \alpha \hat{r}(\theta_1)) = u(W_0 + \alpha \theta_1 + (1 - \alpha)\theta_2 - \frac{1}{2} \alpha^2 \rho \sigma^2)
      \]

      \[
      \Rightarrow \theta_1 \geq \alpha \theta_1 - \frac{1}{2} \alpha^2 \rho \sigma^2 + (1 - \alpha)\theta_2
      \]

      \[
      \Rightarrow (1 - \alpha)(\theta_2 - \theta_1) < \frac{1}{2} \alpha^2 \rho \sigma^2
      \]

    **Proposition.** When the level of projects’ self-financing is observable, there is a continuum of signalling equilibria, parameterized by a number fulfilling \( \frac{\alpha^2}{1 - \alpha} \geq \frac{2(\theta_2 - \theta_1)}{\rho \sigma^2} \) and characterized by a low price \( P_1 = \theta_1 \) for entrepreneurs who do not self-finance and a high price \( P_2 = \theta_2 \) for entrepreneurs who self-finance a fraction of their projects.

  - In the above equilibrium:

    * \( \theta_1 \)-entrepreneurs get the same outcome as in the full-information case

    * \( \theta_2 \)-entrepreneurs get lower utility, i.e. \( u(W_0 + \theta_2 - \frac{1}{2} \rho \sigma^2 \alpha^2) \) instead of \( u(W_0 + \theta_2) \). The difference in the income is \( C = \frac{1}{2} \rho \sigma^2 \alpha^2 \) : informational cost of capital

    * \( (4) \) is preferred by the high type since it minimizes the retention of shares and avoid mimicing.
0.5.3 Coalition of Borrowers

- In the optimal, the cost of capital equals $C = \frac{1}{2} \rho \sigma^2 \alpha^2(\sigma)$ where () satisfying $\frac{\alpha^2}{1-\alpha} = \frac{2(\theta_2 - \theta_1)}{\rho \sigma^2}$.

  - Note that the cost of capital is increasing with the variance of the return

    * $\alpha(\sigma)$ decreases in $\sigma$ since the agent wants to retain less equity when the project is more risky; if the agent is risk averse, then there is a tradeoff between signaling and risk transfer

      - Rewrite $\frac{\alpha^2}{1-\alpha} = \frac{2(\theta_2 - \theta_1)}{\rho \sigma^2}$ as $\sigma^2 \alpha^2 = \frac{2(\theta_2 - \theta_1)}{\rho} (1 - \alpha)$ and observe that an increasing $\alpha(\sigma)$ imply an increasing $\alpha^2(\sigma)$.

- Coalitions of Borrowers

  * Let $N$ identical entrepreneurs of type $\theta_2$ form a partnership and collectively issue securities in order to finance their $N$ projects

  * If the individual returns are independently distributed, the expected return per project is still $\theta_2$ but the variance per project is now $\frac{\sigma^2}{N}$

  * Given the above remark, coalitions of borrowers can do better than individual borrowers

0.6 Diamond (1984)

0.6.1 Delegated Monitoring Theory

- Monitoring, in broad sense, means

  - Screening project a priori in a context of adverse selection $\implies$ ex-ante monitoring

    * Preventing opportunistic behavior of a borrower during the realization of a project (moral hazard) $\implies$ interim monitoring

    * Punishing or auditing a borrower who fails to meet contractual obligations (costly state verification) $\implies$ ex-post monitoring

      - Interim monitoring: when a bank meet agent at which she lends, she can monitor the behavior of the entrepreneur and see at her interest.

      - Preventing opportunistic behavior: the bank can monitor the hidden actions of the entrepreneur better than the market.

  - Delegated monitoring theory of financial intermediation

    * Financial intermediaries may have comparative advantage in those monitoring activities
Necessary conditions for this theory to work

- Scale economies in monitoring: a bank typically finances many projects
- Small capacity of investors: each project needs the funds of several investors
- Low costs of delegation: the cost of monitoring the monitor (FIs) has to be low

0.6.2 Model:

- n identical risk-neutral firms seek to finance projects
  - Initial investment normalized to 1
    - Returns are identically and independently distributed
    - Cash flow $\tilde{y}$ of the investment is unobservable to lenders
    - $y = 0$ is possible

- (Small) Investors
  - Each investor owns only $\frac{1}{m} \Rightarrow m$ investors are needed for financing one project.

- Monitoring
  - Lenders are able to observe the realized cash flow by paying a monitoring cost $K$
  - interim monitoring: the investors pay $K$ at date 0 and observe $\tilde{y}$; i.e. decision to pay the cost $K$ and the payment are made ex-ante and the observation is made ex-post

- Audit:
  - ex-post: $\hat{y}$ is realized, firm report $\hat{y}$ and investors can choose to audit; i.e. the decision to audit, the payment and the observation are made ex-post
    - Can audit with a cost $\gamma$ and $\begin{cases} \text{inflict a penalty} & \text{if } \hat{y} \neq \tilde{y} \\ \text{do nothing} & \text{if } \hat{y} = \tilde{y} \end{cases}$

- Direct lending with monitoring (no monitoring coordination):
  - Each investor monitors the firm he has financed $\Rightarrow$ Total monitoring cost is $mnK$
    - $m$ investors monitor $n$ projects at a cost $K$ per project monitor per investor
* there is no coordination of the monitoring, no pooling of the monitoring contract

\[ \text{cost for the investors to monitor the bank} + \text{cost to monitor the projects} \]

\[ mnK > \text{cost for the investors to directly monitor the projects} \]

**Delegating the monitoring to a bank:** how to incentivize the bank to repay depositors?

- If depositors monitor the bank \( \implies \text{total cost is} \) \[ mnK + nK \]
- If depositors sign a debt contract \( D \) (deposit contract) with the bank:
  - Deposit rate: \( r_D \)
  - Auditing the bank in case of non payment: unit cost of audit is \( \implies \text{total cost of audit if the bank has} \) \( n \) borrowers \( C_n = n\gamma P(\hat{y}_1 + \hat{y}_2 + \ldots + \hat{y}_n < (1 + r_D) \cdot n) \)
  - Audit if \( \hat{y} < D \)
  - Assuming \( K < C_1 \) \( \implies \text{the bank will choose to monitor} \) his borrowers instead of signing debt contracts with them if there is only one project
· **Question**: whether \( \frac{nK}{n} + \frac{C_n}{n} < mnK \) \( \Leftrightarrow \)

\[ K + \frac{C_n}{n} < mK? \]

**Proposition.** If monitoring is efficient \((K < C_1)\), investors are small \((m > 1)\), and investment is profitable \(\mathbb{E}[\tilde{y}] \geq 1 + r + K)\), financial intermediation \((\text{delegated monitoring})\) dominates direct lending as soon as \(n\) is large enough \((\text{diversification})\)

**Proof.** In the equilibrium, the deposit rate \(r_n^D\) promised to depositors by a bank with \(n\) borrowers is determined by

\[ \mathbb{E}[\min(\tilde{Z}_n, 1 + r_n^D)] = 1 + \frac{C_n}{n} \]

where \(\tilde{Z}_n = \frac{1}{n}(\tilde{y}_1 + \tilde{y}_2 + \ldots + \tilde{y}_n) - K\)

If \(n \to \infty\), \(\tilde{Z}_n \to \mathbb{E}[\tilde{y}] - K\). Since \(\mathbb{E}[\tilde{y}] \geq 1 + r + K\), we have \(\lim_{n \to \infty} r_n^D = r\). Therefore

\[ \lim_{n \to \infty} \frac{C_n}{n} = \lim_{n \to \infty} \mathbb{P}(\tilde{Z}_n + K < 1 + r) = 0 \]

(where \(\frac{C_n}{n} \to 0\) implies that the banks have increasing return to scale) \(\square\)

- The proposition implies that as \(n \to \infty\) the bank never audit

- **Discussion:**

  - Adverse selection model: diversification is good since it reduces the informational cost
  - Shortcoming:
    * This model suppose increasing return to scale, but in this case what explain the presence of several banks?
    * Banks can secretly choose the correlation between projects so why are they choose independent projects; it is not natural giving a possibility of risk shifting.

  - In Diamond (1984), the cost of monitoring is assumed to be fixed and not depend on the number of project to be monitored
    * Cerasi and Daltung (2000) relaxes this assumption
      · Considering the internal organization of banks, monitoring becomes more and more costly as the size of the bank increases \(\Rightarrow\) trade-off of increasing size: increasing the diversification possibilities but also the cost of monitoring \(\Rightarrow\) this trade-off implies the optimal size of the bank
    * Disciplining role of demandable deposits
      · The threat of a bank run is a good commitment to force the bank to monitor projects. It explains why banks deposits are highly liquid.
0.7 Banks Loans vs. Direct Debts

0.7.1 Introduction

- Analyzing the choice between direct finance (issuance of debts on the financial market) and intermediate finance (bank loans)
  - Direct debt is less expensive than bank loans ⇒ only firms that cannot issue debts select bank loans
    * Bank loan decrease the informational asymmetry, but increases the costs
  - Explanation for the coexistence of the two types of finance
    * Moral hazard prevents firms without enough assets from obtaining direct finance

0.7.2 Model of Credit Market with Moral Hazard

- Firms need to finance investment projects
  - Project’s size is normalized to 1
  - Two available technologies

<table>
<thead>
<tr>
<th></th>
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<tbody>
<tr>
<td>Good Project</td>
<td>G</td>
<td>0</td>
<td>π_G</td>
</tr>
<tr>
<td>Bad Project</td>
<td>B</td>
<td>0</td>
<td>π_B</td>
</tr>
</tbody>
</table>

- Assumptions:
  - B > G : this condition will cause an asset substitution problem
  - π_GG > 1 > π_BB
  - Riskless interest normalized to 0

- Moral Hazard
  - The success of the investment is verifiable but Not the firm’s choice of technology nor the return
    * the non observability of the return will be required for risk shifting

- Financial contract specifying
  - A fixed payment R in case of success, 0 in case of failure

- Equilibrium in the absence of monitoring
- Good project is chosen iff:
\[
\pi_G(G - R) \geq \pi_B(B - R)
\] (1)
which is equivalent to:
\[
R \leq R_C = \frac{\pi_G G - \pi_B B}{\pi_G - \pi_B}
\] (2)

- Break-even condition for competitive investors: \( \pi(R) \cdot R = 1 \) (if there is no discounting of the future) where \( \pi(R) \) is the probability of receiving the repayment \( R \)
- (1) implies that the manager prefers good to bad projects
- By (2), we have that if \( R \geq R_C \) then the firm has too much debt ⇒ she will take too much risk
  * \( R < R_C \) ⇒ \( \pi(R) = \pi_G \) and \( R = \frac{1}{\pi_G} < G \)
  * \( R \geq R_C \) ⇒ \( \pi(R) = \pi_B \) and \( R = \frac{1}{\pi_B} \geq B \), there is no credit, we know that the entrepreneur will take a bad project

\* Equilibrium in the absence of monitoring
- From (2), we have:
\[
\pi(R) = \begin{cases} 
\pi_G & \text{if } R \leq R_C \\
\pi_B & \text{if } R \geq R_C 
\end{cases}
\]
- Because \( \pi_B B < 1 \) and \( R < G < B \), no competitive equilibrium where \( R > R_C \) can exist
- Equilibrium where \( R \leq R_C \):
\[
\pi_G R = 1 \iff R = \frac{1}{\pi_G} \leq R_C
\]

\* Equilibrium with monitoring
- By monitoring, banks can prevent firms from using the bad technology
  * Monitoring is costly: the monitoring cost is \( C \)
  * Banks loans increase cost, but decrease the asymmetry of information
- Payment to banks \( R_m \) must satisfy
\[
\pi_G R_m \geq 1 + C
\]
equivalent to
\[
R_m \geq \frac{1 + C}{\pi_G} \geq \frac{1}{\pi_G}
\]
Obviously, monitoring finance (bank loans) are more expensive than non-monitoring finance (direct debt).

**Equilibrium with monitoring**

- Bank lending is possible at equilibrium if

\[
\begin{align*}
\frac{1+\pi}{\pi_G} < G & \quad \text{if } \pi_G \geq \frac{1}{R_C} \\
\frac{1}{\pi_G} \geq R_C & \quad \Leftrightarrow \quad \frac{1+\pi}{\pi_G} < \frac{1}{R_C}
\end{align*}
\]

**Proposition.** Assume that the monitoring cost \( C \) is small enough so that \( \frac{1}{R_C} \geq \frac{1+\pi}{\pi_G} \), there are three possible regimes of the credit market at equilibrium:

- if \( \pi_G \geq \frac{1}{R_C} \), firms issue direct debt at a rate \( R = \frac{1}{\pi_G} \)
- if \( \pi_G \in \left[ \frac{1+\pi}{\pi_G}, \frac{1}{R_C} \right] \), firms borrow from banks at a rate \( R_m = \frac{1+\pi}{\pi_G} \)
- if \( \pi_G < \frac{1+\pi}{\pi_G} \), no trade at the equilibrium, the market collapses

\[
\frac{1}{R_C} \geq \frac{1+\pi}{\pi_G} \Leftrightarrow \frac{\pi_G}{\pi_G - \pi_B} \geq \frac{1+\pi}{\pi_G} \text{ is true if } C \text{ is small enough}
\]

### 0.7.3 Reputation

- Diamond (1991)

- By building a reputation, firms can issue direct debt instead of using bank loans
- An extension of the previous model to the dynamic case

#### 0.7.3.1 Diamond (1991)

- Two dates \( t = 0, 1 \)

- Heterogeneous firms

  - Only fraction \( f \) of them has the choice between the two technologies
  - The rest has access only to the bad one

  - Fraction \( f \) can choose G or B and fraction \( 1 - f \) can only choose B

- If all firms borrow from banks at \( t = 0 \)

  - For firms that were successful at date 0

    - Let \( \pi_S \) the probability of repaying \( R_S \) at date 1 conditionally on success at date 0

    - Using Bayes’ formula:

\[
\pi_S = \frac{\mathbb{P}(S_{t=1} \cap S_{t=0})}{\mathbb{P}(S_{t=0})} = \frac{f\pi_G^2 + (1 - f) \cdot \pi_B^2}{f\pi_G + (1 - f) \cdot \pi_B}
\]
- For firms that were unsuccessful at date 0
  * Let $\pi_U$ the probability of repaying $R_U$ at date 1 (conditional on being unsuccessful at $t = 0$)
    \[
    \pi_U = \frac{f \cdot \pi_G \cdot (1 - \pi_G) + (1 - f) \cdot \pi_B \cdot (1 - \pi_B)}{f \cdot (1 - \pi_G) + (1 - f) \cdot (1 - \pi_B)}
    \]
- Depending on the result at date $t = 0$, the banks update their beliefs in order to choose the project at date $t = 1$
  * At $t = 0$ all firms use bank debt
  * At $t = 1$:
    - firms that were successful at $t = 0$ choose direct finance
    - firms that were unsuccessful at $t = 0$ choose bank debt

- If all firms borrow from banks at $t = 0$
  - $\pi_0$: unconditional probability of success at $t = 0$
    \[
    \pi_0 = f \cdot \pi_G + (1 - f) \cdot \pi_B
    \]
  - We have $\pi_U < \pi_0 < \pi_S$
    \[
    \pi_S \geq \pi_0 \iff f \cdot \pi^2_G + (1 - f) \cdot \pi^2_B \geq (f \cdot \pi_G + (1 - f) \cdot \pi_B)^2
    \]
    \[
    \iff f \cdot \pi^2_G + (1 - f) \cdot \pi^2_B \geq f^2 \cdot \pi^2_G + (1 - f)^2 \cdot \pi^2_B + 2f \cdot (1 - f) \cdot \pi_G \cdot \pi_B
    \]
    \[
    \iff (\pi_G - \pi_B)^2 \geq 0
    \]
- $R^0_C$: critical level of debt above which strategic firms choose the bad project at $t = 0$ is defined by
  \[
  \pi_B \left( B - R^0_C + \delta \cdot \pi_G \cdot (G - R_S) \right) + (1 - \pi_B) \cdot \left( \delta \cdot \pi_G \cdot (G - R_U) \right)
  \]
  payment if success at both dates
  \[
  = \pi_G \left( G - R^0_C + \delta \cdot \pi_G \cdot (G - R_S) \right) + (1 - \pi_G) \cdot \delta \cdot \pi_G \cdot (G - R_U)
  \]
  payment if success at date 2 only
  (3)
  If (3) $\geq$ (4), then direct finance is impossible
  \[
  \iff R^0_C = R_C + \delta \cdot \pi_G \cdot (R_U - R_S)
  \]
  static cutoff
  the threshold under which the tradeoff is not possible is higher in the dynamic setting; the future payments incentivize the firm to adopt a good behavior today.

- Reputational concern push firms to behave

**Proposition.** Assume that $\pi_0 \leq \frac{1}{R^C_C}$, $\pi_S \geq \frac{1}{R^C_C}$, $\frac{1}{R^C_G} \geq \pi_U \geq \frac{1 + C}{\pi_U}$ the equilibrium of the two periods model is characterized as follows
  At $t = 0$, all firm borrow from banks at a rate $R_0 = \frac{1 + C}{\pi_U}$
  At $t = 0$, successful firms issue direct debt at a rate $R_S = \frac{1}{\pi_S}$ whereas the rest borrow from banks at a rate $R_U = \frac{1 + C}{\pi_U} \geq R_0$
0.7.3.2 Financial Capital - Holmstrom and Tirole (1997)

- Three types of agents, all are risk neutral
  - Firms (f): borrowers
  - Banks (m): monitors and informed
  - Depositors (u): uninformed investors

- Two types of project:

<table>
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<tr>
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<td>Good Project</td>
<td>R</td>
<td>0</td>
<td>( p_H )</td>
<td>0</td>
</tr>
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<td>Bad Project</td>
<td>R</td>
<td>0</td>
<td>( p_L )</td>
<td>B</td>
</tr>
</tbody>
</table>

- Initial investment for all projects equal \( I \)
  - the investment size is fixed

- Assumptions:
  - \( p_H \geq p_L \)
  - Only good project has a positive NPV: \( \frac{p_H R}{1+r} - I \geq 0 \geq \frac{p_L R + B}{1+r} - I \)

- Monitoring technology of banks
  - Reducing the private benefit from B to b; \( B \geq b \)
  - Costs to banks: \( C \)

- There are two layers of moral hazard in this model:
  - one for the manager which needs to be incentivized to choose the good project (which will cause her to give up private benefit B)
  - one for the bank which needs to be incentivize to monitor the project; the bank’s depositors are assumed not to be able to observe whether the bank monitors or not.

* N.B.: Limited Liability (LL) protects the bank and the entrepreneur here. Recall the typical tradeoff between ensuring the agent (the manager in the first bullet above, and the bank in the second) and incentivizing the agent. Generally, risk-neutrality implies the agent doesn’t care about insurance, which would seem to erase the conflict between insurance and incentives. The key here is LL. LL means that the agent cannot be punished - it exogenously creates a sort of insurance for the agent, whether anyone likes it or not.
• Firms are heterogeneous
  - They have different initial levels of capital $A$
  - The distribution of capital among the population of firms is represented by the cumulative function $G(.)$

• Banks’ capital
  - $K_M$ denotes the aggregate capital of banks

• Direct lending
  - Financial contract specifying
    * $I_U$ Borrowed amount
    * $R_U$ Repayment to uninformed investors
  - The agent (manager’s) incentive compatibility condition (ICC) says that the payoff to taking the good project has to outweigh the payoff to taking the bad one:
    \[
    p_H(R - R_U) \geq p_L(R - R_U) + B
    \]
    \[
    \iff \Delta p(R - R_U) \geq B
    \]
    \[
    \iff R_U \leq R - \frac{B}{\Delta p}
    \]
  - But the investors need to receive a sufficient return to make them want to extend the loan in the first place: the individual rationality condition (IR) for investors is:
    \[
    p_H R_U \geq (1 + r) \cdot I_U
    \]
  - Which says the expected (they are risk neutral) return is at least equal to the initial investment times the required gross rate of return.
    \[
    \iff I_U \leq \frac{p_H R_U}{1 + r} \leq \frac{p_H}{1 + r} \left( R - \frac{B}{\Delta p} \right)
    \]
  - where $R - \frac{B}{\Delta p}$ is the largest value that the uninformed investor’s return, $R_U$ can take by the third expression up (If this did not hold the manager/entrepreneur’s incentive constraint would be violated).
  - Therefore (reverting to the old notation, $I_U = I - A$) direct lending is possible iff
    \[
    I - A \leq \frac{p_H}{1 + r} \left( R - \frac{B}{\Delta p} \right) \iff A \geq A(r) = I - \frac{p_H}{1 + r} \left( R - \frac{B}{\Delta p} \right)
    \]
where an increase in $A \iff$ decrease of the level of the limited liability. The rightmost expression is the minimum pledgeable income $\bar{A}$ the entrepreneur must contribute to obtain financing from uninformed investors. We can see that the uninformed investors would invest less when $A$ increases since they need to break even.

- We might ask, can the manager finance the project if the minimum pledgeable income $\bar{A}$ is more than $A$ the amount the entrepreneur has? This brings us to...

- Intermediated lending

  - For firms (entrepreneurs, managers) that don’t have enough capital for issuing direct debt (this is the debt from uninformed investors, we can think of this as a bond issued in the market to multiple bondholders) $\implies$ mixed borrowing. Recall that $A$ is distributed $A \sim G(a)$. Some of the firms have $A \leq \bar{A}$. Suppose they borrow some money from the uninformed investors, and the rest from banks. We denote:
    
    * $I_U$ borrowed from investors,
    * $I_M$ borrowed from banks,
    * $\ell$ Self-financed.

  - We can write the new incentive compatibility constraint (ICC) for firms/entrepreneurs:

    $p_H(R - R_U - R_M) \geq p_L(R - R_U - R_M) + b$

    $\iff R - R_M - R_U \geq \frac{b}{\Delta p}$

    $\iff R \geq R_M + R_U + \frac{b}{\Delta p}$

    or

    $R_M \leq R - R_U - \frac{b}{\Delta p}$.

    Note that in fact this is the same constraint as usual, since the entrepreneur’s (borrower’s) portion of the return, denoted $R_B$, is equal to $R_B = R - R_U - R_M$, so this constraint could also be written in the usual way, as $R_B \geq b/\Delta p$.

  - We can also write the new ICC for banks, where $C$ is the cost of monitoring and $b \leq B$ is the associated lower private benefit that the entrepreneur can obtain if they choose the bad project (the intuition is that the entrepreneur cannot extract as great a private benefit as they otherwise would if they are operating under the watchful eye of the bank’s monitoring):

    $p_H R_M - C \geq p_L R_M$
(must hold for the banks to believe it is worthwhile for them to monitor)

\[ R_M \geq \frac{C}{\Delta p} \]

but the last expression for the firm/entrepreneur’s ICC given above, when it holds with equality, defines the minimum value that can be taken by \( R_M \). If it is true for the minimal value, it must be true for the whole range of values \( R_M \) can take, so we can substitute:

\[ R - R_U - \frac{b}{\Delta p} \geq \frac{C}{\Delta p} \]

\[ \iff R_U \leq R - \frac{b + C}{\Delta p} \]

- IR for banks: given a rate of return required by banks,

\[ \beta I_M = p_H R_M, \]

(which holds with equality by the assumption that banks are competitive)

\[ \iff I_M = \frac{p_H R_M}{\beta}. \]

Using the expression \( R_M \geq \frac{C}{\Delta p} \) from the bank’s ICC,

\[ \frac{p_H}{\beta} R_M \geq \frac{p_H}{\beta} \frac{C}{\Delta p}, \]

or \( I_M \geq \frac{p_H C}{\Delta p \beta} \), must hold.

- Finally, the IR for investors, taking the expression for the minimal value that \( R_U \) can take that we defined above by the ICC inequality constraint for banks, can be written

\[ I_U \leq \frac{p_H R_U}{1 + r} \leq \frac{p_H}{1 + r} \left( R - \frac{b + C}{\Delta p} \right). \]

- We have the financing condition

\[ I - A \leq I_U + I_M \]

which says that the contributions of investors must together be sufficient to cover the project’s financing needs

\[ \iff A \geq I - I_U - I_M \]

\[ \iff A \geq I - I_U - I_M \]
where \( I_U \) and \( I_M \) are the lowest values that these the investor’s and bank’s contributions to the project can take without violating the IR or investors and the the IR of banks respectively. We substitute these, which we derived above:

\[
\iff \ A \geq I - \frac{p_H}{1 + r} R_U - \frac{p_H}{\beta} C \Delta p
\]

and conclude by writing the expression in terms of the minimal value taken by \( A, A \), as a function of the investors required rate of return and the bank's required rate of return.

\[
A \geq A(r, \beta) = I - \frac{p_H}{1 + r} \left( R - \frac{b + C}{\Delta p} \right) - \frac{p_HC}{\Delta p\beta}
\]

\[
\iff \ A(r, \beta) = I - \frac{p_H}{1 + r} \left[ R - \frac{b + C \left( 1 - \frac{1 + r}{\beta} \right)}{\Delta p} \right]
\]

- We can compare this to the expression we had previously, where financing was done directly by uninformed investors, without the presence of banks:

\[
A_{directlending}(r, \beta) = I - \frac{p_H}{1 + r} \left[ R - \frac{B}{\Delta p} \right]
\]

A comparison of these two expressions shows that banking is efficient whenever \( A(r, \beta) \leq A_{directlending}(r, \beta) \), which is true whenever

\[
b + C \left( 1 - \frac{1 + r}{\beta} \right) < B.
\]

- We note that in the economy just modeled, assuming the distribution \( A \sim G(s) \), when banks exist, some firms would get financed that would not otherwise be financed if there was only the market for direct lending.

- Think about what would happen if \( \beta > 1 + r \), that is that the rate of return required by banks is greater than that required by the market of uninformed investors. Analysing the above expression gives

\[
b + C \left( 1 - \frac{1 + r}{\beta} \right) < B
\]

\[
\iff B - b > C \left( 1 - \frac{1 + r}{\beta} \right)
\]
where the factor multiplying $C$ is less than 1. (MJM: Not sure what this implies... The professor said that banks don’t monitor, which would be an interesting way of modelling the crisis. However, it seems that it is more complicated than a simple case of $\beta > 1 + r$, because it depends on the cost $C$ as well...)

- Determination of $\beta$ (the rate of return required by banks)
  
  * The equality of supply and demand on loans market implies that the amount of capital provided by banks exactly equals the demand for capital from firms that have assets less than $\bar{A}$ but more than $A$. This is just the mass of such firms multiplied by the amount of financing required.

\[
K_M = [G(\bar{A}(r)) - G(A(r, \beta))] \cdot I_M
\]

where $G(\bar{A}(r)) - G(A(r, \beta))$ is the number of firms who borrow from banks.

**Proposition.** At equilibrium, only well-capitalized firms $(A \geq \bar{A}(r))$ can issue debt. Firms with intermediate capitalization $(A(r, \beta) \leq A < \bar{A}(r))$ borrow from banks and undercapitalized firms $(A < A(r, \beta))$ cannot invest.

The graph above represents the distribution of firm’s initial cash assets, and what category of financing they receive. Taken from the course slides.

0.7.3.3 Bank Loans vs. Direct Debt - Bolton and Freixas (2000)

- $t = 0, 1, 2$ three dates
• Risk neutrality

• Firms have an investment project
  
  \[ t = 0 \], outlay of 1
  
  \[ t = 1 \], project return in case of success = \( y \), with probability \( p \), in case of failure = 0, with complementary probability
  
  * then decision is made whether the liquidate. If so, the payoff is \( A \).

  \[ t = 2 \], if the firm wasn’t liquidated, project return in case of success = \( y \), with probability \( p' \), in case of failure = 0, with complementary probability

<table>
<thead>
<tr>
<th></th>
<th>Success</th>
<th>Failure</th>
<th>Probability(success)</th>
<th>Probability(failure)</th>
</tr>
</thead>
<tbody>
<tr>
<td>( t = 1 )</td>
<td>( y )</td>
<td>0</td>
<td>( p )</td>
<td>( A )</td>
</tr>
<tr>
<td>( t = 2 )</td>
<td>( y )</td>
<td>0</td>
<td>( p' )</td>
<td>0</td>
</tr>
</tbody>
</table>

\( p \sim [p, 1] \) with \( p < 1/2 \), and is publicly observable (think of it as something like a credit rating.)

\( p' \) is private the firm’s private information, observable at \( t = 0 \), and can take the two values \{0,1\}. For bad firms it is 0, and for good firms 1. We might expect that this will cause adverse selection.

• \( p, p' \) are drawn independently
  
  – At date 0, creditor’s prior beliefs about the value of \( p' \) are that \( p' = 1 \) with probability \( v \)

  \[ \mathbb{E}(p') = v \]

  – the true value of \( p' \) is revealed at time 1 to the bank and at time 2 to other securityholders

• Firm’s 3 financing choice, assuming no combinations of financial instruments

1. Bond financing
  
  – repay fixed amount \( R \) at date 1, in case of success, and nothing at date 2. if there is a default at date 1, firm is declared bankrupt and is liquidated, yielding a return of \( A \).

  – Assumption (says that in expectation, the project will pay off enough at date 1 to repay the date 0 investment of 1 unit of currency):

\[ \mathbb{E}(\text{projectcashflow}_{t=1}) > 1 \]

\[ \iff p y + (1 - p) A > 1 \]
The zero-profit condition for investors:
\[ pR + (1 - p)A = 1 \]
and the expected profit of good firms financed with bonds (B):
\[ \Pi_B = \Pi_B^{t=1} + \Pi_B^{t=2} \]
\[ \Pi_B = p(y - R) + py \]
together imply that
\[ \Pi_B = p \left[ y - \left( \frac{1 - (1 - p)A}{p} \right) \right] + py \]
\[ \iff \Pi_B = 2py - 1 + (1 - p)A \]

2. Equity financing
- a share \( a \in [0, 1] \) of the cashflows sold to investors
- the zero profit condition for outside shareholders:
\[ a(py + vy) = 1 \]
and the expected profit of good firms financed with equity (E):
\[ \Pi_E = \Pi_E^{t=1} + \Pi_E^{t=2} \]
\[ \iff \Pi_E = (1 - a)(py + y), \]
together give
\[ \iff \Pi_E = \left( 1 - \frac{1}{py + vy} \right) (py + y) \]
\[ \iff \Pi_E = \left( y - \frac{1}{p + v} \right) (p + 1) \]

3. Bank debt
- repay \( \hat{R} \) at time 1 in case of success and nothing at time 2. If there is a default at time 1, there is a renegotiation and the bank is able to extract the whole surplus at time 2.
- zero-profit condition for banks where \( \gamma \) is the intermediation cost
\[ 1 + \gamma = \hat{R}p + (1 - p)[vy + (1 - v)A] \]
\[ \iff 1 + \gamma = \hat{R}p + (1 - p)[A + v(y - A)] \]
expected profit of the firm when financed with a bank loan is
\[ \Pi_{BL} = p(y - \hat{R}) + py \]
Together, the result will be that the preference between these three types of financing will be determined by what we termed the credit risk $p$ and the dilution costs $v$. 
Part II

The Industrial Organization Approach
0.8 Introduction
The Industrial Organization (IO) view of banking reduces a bank to a normal firm. In this approach our perspective is that a bank is a firm in the business of producing deposits and loans.

0.9 Perfect Competitive Banks

0.9.1 Financial Sector
- $N$ banks indexed by $i = 1, 2, 3, ..., N$
- Production of deposits ($D$) and loans ($L$).
  - There is a cost function $C(D, L)$ common to all banks. Assume $C''(.) > 0$ (convexity) and twice-differentiability
- Bank $n$’s balance sheet looks like

<table>
<thead>
<tr>
<th>Assets</th>
<th>Liabilities</th>
</tr>
</thead>
<tbody>
<tr>
<td>$R_n = C_n + M_n$ (Reserves)</td>
<td>$D_n$ (Deposits)</td>
</tr>
<tr>
<td>$L_n$ (Loans)</td>
<td></td>
</tr>
</tbody>
</table>

- where $M_n$ is the bank’s net position on the interbank market and
- $C_n$ is the bank’s cash reserves at the central bank
  - $C_n \geq \alpha D_n$ where $\alpha$ is the coefficient of compulsory reserves

0.9.2 Interaction with the real sector
Comprised of households, firms, and government
The relationship between the different agents can be represented as in the following diagram
0.9.3 Credit Multiplier Approach

- Monetary base $M_0$ is
  \[
  M_0 = \sum_{i=1}^{N} C_n = \alpha \sum_{i=1}^{N} D_n = \alpha D \Rightarrow D = \frac{M_0}{\alpha}
  \]
  and because $D = M_0 + L$ in aggregate,
  \[
  L = M_0 \left( \frac{1}{\alpha} - 1 \right).
  \]

- therefore
  \[
  \frac{\partial D}{\partial M_0} = \frac{1}{\alpha} > 0,
  \]
  \[
  \frac{\partial L}{\partial M_0} = \frac{1}{\alpha} - 1 > 0.
  \]

- In a competitive market, banks take all prices as given:
  - $r_D$ is the interest rate on deposits
  - $r_L$ is loan interest rate
- $r$ is the interest rate on the interbank market

- The profit optimization problem for each bank (we drop the subscript $n$ to avoid clutter)

$$\max_{D, L} \Pi = r_L L + r M - r_D D - C(D, L)$$

s.t. $M = D - L - \alpha D$

yields the FOCs

$$\frac{\partial \Pi}{\partial L} = r_L - r - \frac{\partial C}{\partial L}(D, L) = 0$$

$$\frac{\partial \Pi}{\partial D} = r(1 - \alpha) - r_D - \frac{\partial C}{\partial D}(D, L) = 0$$

These conditions say that each bank will adjust its volume of loans and deposits so that the "intermediation margin," $r_L - r$ and $r(1 - \alpha) - r_D$ equal its marginal management cost, $\partial C/\partial (\cdot)$.

- an increase in $r_D$ implies a decrease in the bank’s demand for deposits, while an increase in $r_L$ means the bank would increase its supply of loans... (while this makes intuitive sense, I need some help understanding the math here, maybe second order conditions?)

- We solve for equilibrium by clearing the interbank market. Assume that each bank $n$,

- $L_n(r, r_L, r_D)$ is the loan supply function

- $D_n(r, r_L, r_D)$ is the deposit demand function

- $I(r_L)$ is the aggregate investment demand of firms

- $S(r_D)$ is the savings function of households

- Then the set of equilibrium interest rates $\{r, r_L, r_D\}$ is determined by the clearing conditions:

$$I(r_L) = \sum_{i=1}^{N} L_n(r, r_L, r_D) \text{ (loans market)}$$

$$S(r_D) = \sum_{i=1}^{N} D_n(r, r_L, r_D) + B \text{ (deposits market)}$$

$$\sum_{i=1}^{N} L_n(r, r_L, r_D) = (1 - \alpha) \sum_{i=1}^{N} D_n(r, r_L, r_D) \text{ (interbank market)}$$

Where $B$ is the net supply of treasury bills. Let’s solve for equilibrium assuming the cost function is linearly separable, for example

$$C(D, L) = \gamma_D D + \gamma_L L.$$
Then the FOCs become
\[
\frac{\partial \Pi}{\partial L} : r_L - r = \gamma_L \\
\implies r_L = r + \gamma_L
\]
\[
\frac{\partial \Pi}{\partial D} : r(1 - \alpha) - r_D = \gamma_D \\
\implies r_D = r(1 - \alpha) - \gamma_D.
\]
• We will input these expressions into the market clearing conditions. The clearing condition for the interbank market
\[
\sum_{i=1}^{N} L_i(r, r_L, r_D) = (1 - \alpha) \sum_{i=1}^{N} D_i(r, r_L, r_D)
\]
gives
\[
\implies I(r_L) = (1 - \alpha)[S(r_D) - B]
\]
into which we substitute the expressions for interest rates we determined in the FOCs above:
\[
I(r + \gamma_L) = (1 - \alpha)[S(r(1 - \alpha) - \gamma_D) - B],
\]
which implicitly determines \( r \).

**Proposition 1.** An increase in \( B \) (the net supply of treasury bills) entails a decrease in loans and deposits. However the absolute values are smaller than in the standard model:
\[
\left| \frac{\partial D}{\partial B} \right| < 1; \quad \left| \frac{\partial L}{\partial B} \right| < 1 - \alpha
\]

If the reserve coefficient \( \alpha \) increases, the volume of loans decreases, but the effect on deposits is ambiguous.

**Proof.** We will prove the first inequality only. The second is left as an exercise for the reader. Use the expression we had which implicitly determines \( r \):
\[
I(r + \gamma_L) = (1 - \alpha)[S(r(1 - \alpha) - \gamma_D) - B],
\]
but
\[
I(r + \gamma_L) = I(r_L) = \sum_{i=1}^{N} L_i(r, r_L, r_D) = (1 - \alpha) \sum_{i=1}^{N} D_i(r, r_L, r_D)
\]

by the market clearing conditions, and since \( \sum_{i=1}^{N} D_i(r, r_L, r_D) = D \), we can write
\[ D = S(r(1 - \alpha) - \gamma_D) - B \]

\[ \Longrightarrow \frac{\partial D}{\partial B} = (1 - \alpha)S'(r) \cdot \frac{\partial r}{\partial B} - 1 \]

and we can compute \( \partial r / \partial B \) by taking the partial derivative of

\[ S(r(1 - \alpha) - \gamma_D) = \frac{I(r + \gamma_L)}{1 - \alpha} + B \]

\[ \Longrightarrow (1 - \alpha) \cdot S'(r) \cdot \frac{\partial r}{\partial B} = \frac{1}{1 - \alpha} \cdot I'(r) \cdot \frac{\partial r}{\partial B} + 1 \]

\[ \iff \frac{\partial r}{\partial B} = \frac{1}{(1 - \alpha)S'(r) - \frac{1}{1 - \alpha} I'(r)} \cdot \frac{1}{1 - \alpha} I'(r) \]

substituting this into our expression for \( \partial D / \partial B \):

\[ \frac{\partial D}{\partial B} = (1 - \alpha)S'(r) \cdot \frac{1}{(1 - \alpha)S'(r) - \frac{1}{1 - \alpha} I'(r)} - 1 \]

\[ = \frac{(1 - \alpha)S'(r) - (1 - \alpha)S'(r) - \frac{1}{1 - \alpha} I'(r)}{(1 - \alpha)S'(r) - \frac{1}{1 - \alpha} I'(r)} \]

\[ = \frac{\frac{1}{1 - \alpha} I'(r)}{(1 - \alpha)S'(r) - \frac{1}{1 - \alpha} I'(r)} > -1 \]

\[ \square \]

0.9.4 Monopolistic Bank (Monti-Klein Model)

A monopolistic bank doesn’t take prices as given.

- Assume an inverse loan demand function \( r_L(L) \)
- and an inverse deposit supply function \( r_D(D) \)
- Assuming the interbank rate \( r \) is given for bank. The bank’s profit optimization is

\[ \max_{D,L} \Pi = (r_L(L) - r)L + [r(1 - \alpha) - r_D(D)]D - C(D, L) \]

which yields the FOCs:

\[ \frac{\partial \Pi}{\partial L} = r'_L(L) + r_L(L) - r - \frac{\partial C}{\partial L}(D, L) = 0 \]

\[ \frac{\partial \Pi}{\partial D} = r(1 - \alpha) - r_D(D) - r'_D(D)D - \frac{\partial C}{\partial D}(D, L) = 0 \]
• Elasticities $\epsilon_L$ and $\epsilon_D$

$$
\epsilon_L = \frac{r_L L'(r_L)}{L(r_L)}
$$

$$
\epsilon_D = \frac{r_D D'(r_D)}{D(r_D)}
$$

• The solution $(r_D^*, r_L^*)$ is characterized by

$$
\frac{r_L^* - r - \frac{\partial C}{\partial L}}{r_L^*} = \frac{1}{\epsilon_L(r_L^*)}
$$

$$
\frac{r(1 - \alpha) - r_D^* - \frac{\partial C}{\partial D}}{r_D^*} = \frac{1}{\epsilon_D(r_D^*)}
$$

• See slides 12-34 for the remainder of this lecture

0.9.4.1 Model

• In the following model, borrowers are also depositors. Each borrower has a fixed loan demand of size $L < 1$. If there are insufficient deposits to cover the aggregate demand for loans, the bank can borrow additional funds from the money market and lend these to borrowers. Likewise (conversely) any excess of deposits over the loan demand can be invested in the money market.

• Transportation cost parameters

  - $t_D$ for deposits, all of which are of size 1
  - $t_L$ for loans, which are of size $L < 1$.

• We can solve for equilibrium when contracts must be independent. For a borrower to prefer $i$ to $i+1$:

$$
(1 + r_L^i) L - t_L(x_i) \geq (1 + r_L^{i+1}) L - t_L \left( \frac{1}{n} - x_i \right)
$$

0.9.5 The Threat of Termination (Bolton and Scharfstein, 1990)

If an audit is impossible, how else can we give a borrower incentives to repay? What could be an incentive device in this case? Bolton and Scharfstein (1990) explores repeat borrower-lender relationships with the “threat of termination” being used to incentivize borrowers. Another example is Jappelli, Pagano, and Bianco (2005) which looks at judicial enforcement through the legal system.

The model:

• Three dates: $t = 0, 1, 2$. 
The production technology allows the borrower to invest \( 1 \) at date \( t \), and at \( t+1 \) the borrower can get binomial cash flow \( \tilde{y} \)

\[
\begin{array}{c|c}
\text{t} & \text{t+1} \\
\hline
-1 & \tilde{y} \\
\end{array}
\]

where

\[
\tilde{y} = \begin{cases} 
  y_H & \text{with probability } p_H \\
  y_L & \text{with probability } p_L = 1 - p_H 
\end{cases}
\]

and where \( \mathbb{E}(\tilde{y}) > 1 \), and \( y_L < 1 \). Cashflows are i.i.d. across dates. They are not observable, so there is moral hazard.

- Entrepreneur has no wealth (\( A = 0 \) in our notation) and is risk-neutral.
- The bank can terminate the loan at \( t = 1 \) if the borrower cannot repay.

- with only 1 period credit rationing, the situation would be inefficient, lender would only expect to get \( y_L \)

Let’s calculate the expected present value of the bank profit. Note that date 2 is the final date. Since \( \tilde{y} \) is not observable this means at date 2 the bank always gets back \( y_L \). Let \( R \) be the amount specified in the contract to be paid to the bank by the entrepreneur at date 1. Then we can write

\[
\pi = -1 + (\text{expected return}_{t=1}) + \mathbb{P}(\text{not liquidated}) \times (-1 + y_L)
\]

\[
\iff \pi = -1 + p_H R + p_L y_L + p_H (-1 + y_L)
\]

\[
\iff \pi = -1 + p_L y_L + p_H (R - 1 + y_L)
\]

\[
\iff \pi = p_H (R - 1) + y_L - 1.
\]

- Write \( IC_{\text{borrower}} \):

\[
p_H (y_H - R) + p_L (0) + p_H [p_H (y_H - y_L)] + p_L (0) \geq p_H (y_H - y_L) + p_L (0) + 0
\]

\[
\iff y_H - R + p_H (y_H - y_L) \geq (y_H - y_L)
\]

\[
\iff -R + p_H (y_H - y_L) \geq -y_L
\]

\[
\iff R \leq y_L + p_H (y_H - y_L)
\]

\[
\iff R \leq p_H y_H + (1 - p_H) y_L
\]

\[
\iff R \leq \mathbb{E}(\tilde{y}).
\]

- The \( PC_{\text{lender}} \):

\[
\pi \geq 0 \iff p_H (R - 1) + y_L - 1 > 0
\]

\[
\iff R \geq 1 + \frac{1 - y_L}{p_H}.
\]
• Combining $IC_{borrower}$ and $PC_{lender}$,

$$1 + \frac{1 - y_L}{p_H} \leq R \leq \mathbb{E}(\tilde{y})$$

$$\iff 1 - y_L \leq p_H[\mathbb{E}(\tilde{y}) - 1]$$

But we might ask: is this renegotiation-proof? The bank will lose (for certain) $1 - y_L$ in the last period, and the borrower gains $p_H(y_H - y_L)$.

• Note: difference between default and renegotiation. Yes, the bank would default if it could, but we assume that contracts are enforceable. BUT, the bank would try to renegotiate. If $p_H(y_H - y_L) < (1 - y_L)$ implies $p_Hy_H + y_L(1 - p_H) < 1$ (negative NPV), the bank might want to just pay directly. But if this were true, the project would have negative NPV.

• Even if it’s a social improvement, there’s no way the borrower can transfer more than $y_L$ to the bank, no renegotiation possible, so the threat of termination is credible.

• “Commitment” here refers to the commitment not to renegotiate, not the commitment not to default. We assume the bank cannot default. Ex. In Townsend paper, we need full commitment.

0.9.6 Judicial Enforcement (Japelli, Pagano, and Bianco, 2005)

The idea of the following model is that the efficacy of the legal system matters. How might the judiciary system impact the terms and availability of credit? Perhaps the cost and length of trials have an impact, maybe the degree of protection given to the borrower determines the borrower’s opportunities to renegotiate the contracted repayment.

Model:

• An economy where the interest rate is normalized to zero and all agents are risk neutral

• Investment project: an initial investment of 1 generates a cash flow:

$$\tilde{y} = \begin{cases} 
  y_H & \text{with probability } p \\
  y_L & \text{with probability } 1 - p 
\end{cases}$$

• success/failure is observable.

• $C$ is amount required to be pledged as collateral

• The “strictness” of the judicial system is modelled using two parameters:

$$\begin{cases} 
  \varnothing_p & \text{recovery rate on the firm’s cash flow} \\
  \varnothing_c & \text{recovery rate on the external collateral} 
\end{cases}$$
• Using these, we can compute the lender the expected repayment to the lender

\[ p \min(R, \varnothing_p y + \varnothing_c C) + (1 - p) \min(R, \varnothing_c C) \]

If you ask for repayment of more than the proportion of what the borrower has left over when they default, they strategically default.

• When \( R \leq \varnothing_p y + \varnothing_c C \) (it is not optimal for the borrower to strategically default), the lender’s expected payoff simplifies to

\[ pR + (1 - p) \min(R, \varnothing_c C) \]

yielding the break even condition (with equality, assuming a competitive lender):

\[ 1 = pR + (1 - p) \min(R, \varnothing_c C). \]

Note that here, default was also assumed to be an option when state \( L \) occurs.

- 3 cases
  
  * High collateral: \( \varnothing_c C > 1 \) (the loan is fully collateralized)
  
  * Too little collateral: \( R \leq \varnothing_p y + \varnothing_c C \), So the maximum repayment the lender can get is:
    \[ p(\varnothing_p y + \varnothing_c C) + (1 - p)\varnothing_c C < 1. \]
    
    Thus we cannot fund the project.
    
    · if repayment of cashflows is well enforced \( \varnothing_p = 1 \), \( NPV > O \), then financed.
    
    * In intermediate case, \( R > \varnothing_c C \), because \( \varnothing_c C \) is smaller than 1

- If there is a huge amount of collateral, no problem, but too little collateral implies credit rationing (projects that would create value cannot be financed). So if repayment of cashflows is well enforced \( \varnothing_p = 1 \), \( NPV > O \), then financed, but in an economy where this is not well enforced, collateral must be large to be financed.

**Strategic Default: Case of Sovereign Debtor**

• A country borrows \( L \) at rate \( r \) to make an investment

• Output produced is \( f(L) \).

• Static profit maximization determines optimal loan \( L_D \):

\[ L_D = \arg \max_L \pi = f(L) - (1 + r)L \]

\[ \iff f'(L_D) = 1 + r. \]
• Dynamic case: (the opportunity cost of default equals the present value of foregone profits):

\[ V(L) = \sum_{t=1}^{\infty} \beta^t [f(L) - (1 + r)L] \]

\[ = \frac{\beta}{1 - \beta} [f(L) - (1 + r)L] . \]

here, the exclusion from the borrowing market (threat of termination) is the only incentive to discourage default. The punishment is being excluded from capital markets forever.

• \( \frac{\beta}{1 - \beta} [f(L) - (1 + r)L] \geq (1 + r)L, \) implies \( \frac{f(L)}{L} \geq \frac{1+r}{\beta}. \)

  - we noted \( \frac{f(L)}{L} < f' \) since it’s a concave function. \( L \leq \hat{L} \) since it’s a concave function.

• When \( \beta \) is large enough, the optimal loan \( L_D \) is feasible.

• But when \( \beta \) is small, \( \hat{L} \) may be smaller than \( L_D \).

• Example: if you are a govt and your utility is your prob of election, you care about the short run... you might, say securitize future revenues or something like this, or postpone future expenses (beta is small...i.e. you discount the future a lot..."Impatience") There is a retrospective voting literature (for example Nordhaus in the 50s)

• Now considering a more complete infinite horizon.

  - Now the country borrows to smooth consumption, since there are exogenous shocks to production (stochastic production process)

  - The objective function of the borrowing country is

\[ U = \mathbb{E} \left( \sum_{t=0}^{\infty} \beta^t u(C_t) \right) \]

  - Borrowing is short-term, and any default is followed by permanent exclusion from future borrowing. Let \( b \in B \) denote the amount of a loan, \( B \) is the set of all possible loans.

  - In the case of default, the continuation utility of the defaulting country is

\[ U_d = \mathbb{E} \left( \sum_{t=0}^{\infty} \beta^t u(\tilde{y}_t) \right) = \mathbb{E} u(\tilde{y}) \frac{1}{1-\beta} \]
Strategic default occurs iff

\[ u(y) + \beta U_d > u(y - R) + \beta V_r \]

where \( R \) is the repayment amount and \( V_r \) is the continuation payoff associated with repaying the current period loan. The above expression is equivalent to

\[ y < \varphi(R) \]

where \( \varphi \) is an increasing function.

- For a given \( y \), the borrowing amount \( b(y) \) solves

\[ V(y) = \max_{b \in B} \{ u(y + b) + \beta \mathbb{E}_y [\max(u(y') + \beta U_d, u(y' - R(b)) + \beta V_r)] \} \]

where \( y' \) is the unknown future output.

- Equilibrium is characterized by \( \{ V(y), b(y), \varphi(R), R(b) \} \) such that the expression for \( V(y) \) above is satisfied where the maximum is obtained for \( b = b(y) \) and \( V_r = \mathbb{E}[V(\tilde{y})] \).

- \( y' < \varphi(R) \) then we have strategic default.

- for all \( b \in B \),

\[ R(b) = \frac{(1 + r)b}{P\{\tilde{y} > \varphi[r(b)]\}} \]

meaning that lenders behave competitively.

- Note that in this setup, the maturity of the loans are each only for one period...(This was the “canonical model” of sovereign borrowing)

Returning to the standard moral hazard setting:

- Consider a static borrower-lender relationship. Both agents are risk-neutral. Suppose a borrower’s return \( \tilde{y} \) is observable. The return’s distribution is affected by some action, say effort.

- let \( f(y, e) \) be the density function of the return \( y \) for a given level of effort \( e \). This action is unobservable by the lender.

- The borrower’s cost of effort is given by \( \psi(e) \), increasing and convex.

- Loan contract specifying \( R(y) \).

- Denote the borrower’s net expected utility by \( V(R, e) \)

\[ V(R, e) = \int (y - R(y))f(y, e)dy - \psi(e). \]
The optimal contract is determined

\[ \max V(R, e^*) e^* \]

\[ e^* = \arg \max V(R, e) \text{ s.t.} \]

\[ 0 \leq R(y) \leq y, \forall y \]

\[ \mathbb{E}[R(y)|e^*] \geq U_0^L \]

where \( U_0^L \) is the minimum return demanded by the lender.

**Proposition 2.** If for all \( e_1 > e_2 \), the likelihood ratio \( \frac{f(y,e_1)}{f(y,e_2)} \) is an increasing function of \( y \) (monotone likelihood ratio (MLR) property), the optimal repayment function is always of the following type

\[ R(y) = \begin{cases} 0 & \text{for } y \geq y^* \\ y & \text{for } y < y^* \end{cases} \]

This is non-increasing! So it’s not a standard debt contract. After a threshold, the optimal repayment drops to 0. The reward is as much as possible in the good state, and the punishment as harsh as possible in the bad state.

**Proof.** We find \( R^* \) and \( e^* \) from the maximization program

admit without proof that we can replace the maximization constraint with the FOC

\[ \frac{\partial}{\partial R} V(R, e^*) = 0 \]

then we can just

\[ \max \int (y - R(y)) f(y, e) dy - \psi(e) \]

s.t. \[ \int (y - R(y)) f_e(y, e) dy = \psi'(e) \]

\[ \int R(y) f_e(y, e) dy = U_0^L \]

\[ \mathcal{L} = \int (y - R) + \lambda(y - R) f_e(y, e) + \mu R f(y, e) dy + [(\mu - 1) f(y, e) - \lambda_e(y, e)] R \]

\[ R f_e(y, e) (\mu - 1 - \lambda_e f_e) \]

\[ R(y) = y \]

\[ f_e/f < \text{someconstant} \]

\[ R(y) = 0 \]

...something here was lost at the very end...
• A little disappointing, but if we constrain the contract to be increasing (R must be increasing) we recover the standard debt contract.

Complete contract vs. Incomplete contracts

• Complete contracts are contracts that are contingent on all future states of nature. Writing complete contracts will improve efficiency, however, it is often too difficult (and also too costly) to describe all possible future contingencies. Incomplete contract theory recognizes this fact.

• An incomplete contract will typically involve some delegation and allocation to one of the contracting parties of the power to choose among a predetermined set of actions. This power is made contingent on the realization of some variable signals. The main insights from incomplete contract models are that the design of contracts should limit the tendency of agents, to whom choice of actions is delegated, to behave inefficiently...

• “No-slavery condition”/inalienability of human capital
  
  – Noncommitment for the entrepreneur not to withdraw human capital from the investment project implies some protable projects will not be funded. The time profile of repayments will be affected by the liquidation value of the project
  
  – Whether this creates credit constraints or not actually depends on the bargaining power of agents

• Simple version of the Hart and Moore model

• Look at two extreme cases
  
  – If creditor has all the bargaining power: if $R_t = y_t$ at each date, it will work, the lender can extract all the surplus, but no efficiency loss
  
  – If entrepreneur has all the bargaining power
    
    * at each date, the remainder of all the repayments must be less than the liquidation value
    
    * Inalienability of human capital + entrepreneur having the bargaining power can create credit constraints. So liquid assets are easier to borrow against. Inalienability is here the source of contract incompleteness. Think of traders, lawyers, etc. May not apply so much to sports.

Myers and Rajan (1998)

• the problem with illiquid assets: the one who operates the assets has the bargaining power. If an asset is very liquid, ex-post liquidation/renegotiation problem is gone, but easy for the agent to trade against a riskier asset (do risk-shifting, asset substitution for example)
They claim it’s a theory of banking... there’s some optimal liquidity level... make balance sheet more illiquid, making illiquid loans with liquid deposits.

Model

- return of project is split between cash $C$ and some continuation value $d$ of the assets in place,
- $C < d$. (we’ll see why we need this...?)
- assets can be liquidated for $\alpha d$, $\alpha$ is a measure of the liquidity of the assets in place.
- Assuming the manager has all the bargaining power implies debt repayment must satisfy
  \[ R \leq \alpha d \]
  (because otherwise the manager prefers to just liquidate and disappear)
- Manager can engage in asset substitution
  - Manager gets $\alpha M d$
  - Investors get 0.
  - If $\alpha d \leq C + d - R$
    Manger doesn’t engage in the asset substitution
- So a debt contract is feasible iff
  \[ R \leq \min(\alpha d, C + d - \alpha d) \]
  - Here we’ve assumed that a liquid asset is easy to “steal” or risk shift, for asset with high liquidation value, it is also
  - If your assets have low value in the hands of the manager, they probably have low value in the hands of someone else
    - Counterexample: Liquid in hand of lenders, but difficult to do asset substitution... (could be a part of complex derivatives?"
      ...
    - Too illiquid: ex post
    - Too liquid: incentive problems..
    - implies maybe there’s an interior solution...

0.9.7 Securitization

- Policy Paper Bank of Japan
Securitization: Transform something that is not tradeable into something that is tradeable (a security). For example, a bank grants a loan, then sells the promises to future cashflows to other investors. They might do this so to free up space in their capital requirement to be able to originate more loans and obtain the fees, for example.

* What are the frictions that make this make sense? (In an M&M world this wouldn’t have a purpose) How it works: Creation of a Special Purpose Vehicle (SPV). Perhaps if you really understand the loan portfolio of the bank, but not the other activities, you might be willing to buy it for less of a discount.

- What do bankers claim? Bankers say it’s because of regulation: securitizing relaxes this. Here we are talking about capital requirement: $E < \alpha L$, and bankers hate issuing equity... can’t take anymore debt, so if want to lend more, need to “recycle” old loans, which they perceive as a cheaper way of financing than issuing new equity.

* Issue: what are the incentives of a bank to monitor if they just originate and sell to the market right after? If we thought banks could produce some info (perhaps from monitoring) that markets couldn’t, then it may be bad for welfare to fo this.

- In most cases, banks keep some a tranche of the pool of loans they securitize to fight against this. We can thing of this as serving a signalling purpose.
- Starting around 2001, many classes of loans that used to be insecured, became securitized.
- When there is a boom then a bust, it means people’s beliefs varied a lot
- people seemed to believe that progress in financial engineering made markets closer to frictionless, complete markets, and the securitization boom improved risk allocation within the world...oops! Now they say the opposite.
- More and more, banks didn’t keep loans for longer than a few months. They became pure brokers, following the originate and distribute model...

- Gary Becker quote:
  thanks to all this dream of owning a home becomes true for many American families.
- Deregulation: Basel II...banks got to use internal models... maybe a bad idea? Self-regulation
∗ What many banks did:
   · create an Special Investment Vehicle (SIV), to which they would then sell a loan, and simultaneously offered a put to the SIV (a guarantee to buy it back at some price) with a maturity of less than 6 months. Because of this short duration, a regulatory loophole meant they didn’t have to include these in their capital requirements.
   · Essential problem: it was thought that information frictions had mostly disappeared, since the cost of info went down due to information technology innovations. Actually the common view now is that securitization destroys a bank’s monitoring incentives
   · Now: Back to arbitrary, tough regulation: every bank has to keep 5% of loan.

∗ But Maybe: There’s nothing inefficient about it! There’s a tradeoff between gains from trade (possibly risk-sharing) and incentives. The increase in defaults in subprime mortgages etc, in the standard model doesn’t imply inefficiency...
   · Paper: high-documentation, but high-FICO (above 620) score borrowers are more likely to default than low-documentation borrowers with high-fico scores...implies banks are willing to lend to borrowers they know are bad since investors will buy the securitized portfolio anyway if the FICO score is high enough. (whereas for below this threshold, low documentation borrowers defaulted more, as you might expect). Nonetheless, doesn’t imply inefficiency, in principle.
   · Plantin models of securitization: 1. Nothing inefficient. 2. Inefficient.

∗ Plantin Parlour JF Paper

− The model is a variant on Holmstrom-Tirole ’97 with firms, banks, and bond-investors that we went through in class. Here, we have one bank and many investors. In this case, firms have constant returns to scale, and outcomes are stochastic and dependant on effort.

− The firm chooses the size of the project ($I$), and how much bank and bond finance it uses.

− There are 2 pieces of private info

   ∗ bank receives a discount factor shock
   ∗ payoff from loan
 CONTENTS

- 3 dates
  
  * $t = 1, 2, 3$ (so 2 periods)

- risk neutral agents, who are protected by limited liability, and don’t discount:

  * F firm, has initial wealth $A$, and a technology transforming 1 into $R$ (or $I$ into $RI$) with probability $p$ and into 0 with probability $1 - p$ if it behaves. If the firm shirks, $p$ becomes $p - \Delta p$, but the firm captures a private benefit $B_F I$

  * B bank can monitor the firm: The firm’s private benefit $B_F I$ is decreased to $B_F I < B_F I$, but bank receives a benefit $B_B I$ if it doesn’t monitor. If the bank shirks, it derives private benefit $B_B I$

    * If the bank monitors, it figures out the date 2 payoff at date 1

  * and investors (bond investors)

- 1st period, there will be a firm effect, and bank’s monitoring

  * 1st case: only socially optimal thing to do is have the bank monitor, and the firm behaves (like if $p = \Delta p$)

  * The bank values liquidity. (this is like Diamond-Dybvig)

    $$ U = E(c_0 + \delta_1 (c_1 + \hat{\delta}_2 c_2)) $$

    where

    $$ \hat{\delta}_2 = \begin{cases} 
    \delta & \text{with probability } q \\
    0 & \text{with probability } 1 - q
    \end{cases} $$

- The firm offers the optimal renegotiation-proof contracts to the bank and investors, but the bank can offer new contracts at date 1. The firm doesn’t care what the bank does once monitoring has taken place at date 1; the bank will trade credit risk with the investors in an ex post optimal way.
if
\[ r = \frac{pq}{1 - p + pq} < \delta \]
where the lhs is the conditional probability of the loan being good conditional on the fact that the bank is selling, then there is no trade in the secondary CRT market. As a real-world example, perhaps during the 90s there was a shock lowering \(\delta\). Let

- * \(I\) total investment, \(I = A + L + M\)
- * \(L\) is the money lent by banks
- * \(M\) from investors
- * \(R_B\) and \(R_I\) are as usual

### No CRT
\[
\text{Max}_{I,L,M,R_F,R_B} \{ pR_F I - A \}
\]
\[
\text{s.t.}
\]
Firm and bank's incentive compatibility:
\[
\begin{align*}
pr_F & \geq b_F \\
\delta_1 E \delta_2 pR_B & \geq B_B
\end{align*}
\]
Market and bank's participation:
\[
\begin{align*}
L & \leq \delta_1 E \delta_2 pR_B I \\
p(R - R_F - R_B) I & \geq M
\end{align*}
\]
- \( I \leq M + A + L \)

### CRT
\[
\text{Max}_{I,L,M,R_F,R_B} \{ pR_F I - A \}
\]
\[
\text{s.t.}
\]
Firm and bank's incentive compatibility:
\[
\begin{align*}
pr_F & \geq b_F \\
\delta_1 pR_B & \geq B_B + \tau R_B
\end{align*}
\]
Market and bank's participation:
\[
\begin{align*}
L & \leq \delta_1 pR_B I \\
p(R - R_F - R_B) I & \geq M
\end{align*}
\]
- \( I \leq M + A + L \)

- In the no CRT (credit risk transfer) and CRT (what if it's a pooling market?)...only thing that changes, no \(E(\delta_2)\) in their preferences, because the bank never keeps the loan through period 2. But now if the bank shirks, it will still be able to sell something worthless. Thus, banks are willing to lend more. But now, more difficult to inventivise banks, because they want to shirk and get the private benefit.

- Bank capital is costly, total cost of bank capital is product of how big their stake is and their total cost per dollar.
- For the right parameter values, in both cases, all constraints are binding, so to solve the model just solve the PC of the bondholders.

\[ I - A - L = pRI - bFI - pRB \]

which says that the “arm’s length finance, \( M = I - A - L \) is equal to the expected payoff to bondholders. And the firm maximizes \( I \):

\[ I_{max} = \frac{1 - pR + bF - pR_{BMI} - L}{1 - pR + bF + \frac{pR_{BMI} - L}{I_{Max}}}. \]

- We can see that the bank’s unit rent,

\[ \frac{pRB_{Max} - L}{I_{Max}} \]

can be decomposed into

\[ \left( \frac{pRB_{Max} - L}{L} \right) \times \frac{L}{I_{Max}} \]

the price decrease with CRT times the quantity increase. And

\[ \frac{pRB_{Max}}{L} = \frac{1}{\delta_1} \]

if there is CRT, and

\[ \frac{pRB_{Max}}{L} = \frac{1}{\delta_1E\delta_2} \]

without it. Likewise,

\[
\begin{cases}
L_{I_{Max}} = B_B \left( \frac{p}{p-r} \right) & \text{if CRT} \\
L_{I_{Max}} = B_B & \text{if no CRT}
\end{cases}
\]

- We can illustrate the results on this interesting graph with 4 areas: no strong connection between when it is liquid and when it is efficient.
- Interpretation. Interpret $\delta$ as being dependent on $p$: we may wish to have a capital requirement that depends on the riskiness of the loans.

- Note that the conditions under which liquidity rises and the conditions under which liquidity is efficient are NOT THE SAME.

  * Inefficient innovation may take place while desirable innovation may not. Liquidity is most likely to rise for high rated names, for which it is inefficient. Names which are rated too low do not trade, though it would be desirable.

- If the rise of CRT during the 90s was driven by a fall in $\delta$ then steeper pricing schedule of loans, but not bonds, the fraction of bank finance is more sensitive to credit rating.